XIX GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN
The final round. First day. 8 form.
Ratmino. 2023. July 30.

1. Let ABC be an isosceles obtuse-angled triangle, and D be a point on its base AB such that AD equals to the circumradius of triangle BCD. Find the value of $\angle ACD$.

2. The bisectors of angles A, B, and C of triangle ABC meet for the second time its circumcircle at points A_1, B_1, C_1 respectively. Let A_2, B_2, C_2 be the midpoints of segments AA_1, BB_1, CC_1 respectively. Prove that the triangles $A_1B_1C_1$ and $A_2B_2C_2$ are similar.

3. The altitudes of a parallelogram are greater than 1. Does this yield that the unit square may be covered by this parallelogram?

4. Let ABC be an acute-angled triangle, O be its circumcenter, BM be a median, and BH be an altitude. Circles AOB and BHC meet for the second time at point E, and circles AHB and BOC meet at point F. Prove that $ME = MF$.

XIX GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN

The final round. Second day. 8 form.

Ratmino. 2023. July 31.

5. The median CM and the altitude AH of an acute-angled triangle ABC meet at point O. A point D lies outside the triangle in such a way that $AOCD$ is a parallelogram. Find the length of BD, if $MO = a$, $OC = b$.

6. For which n the plane may be paved by congruent figures bounded by n arcs of circles?

7. The bisector of angle A of triangle ABC meet its circumcircle ω at point W. The circle s with diameter AH (H is the orthocenter of ABC) meets ω for the second time at point P. Restore the triangle ABC if the points A, P, W are given.

8. Two circles ω_1 and ω_2 meeting at point A and a line a are given. Let BC be an arbitrary chord of ω_2 parallel to a, and E, F be the second common points of AB and AC respectively with ω_1. Find the locus of common points of lines BC and EF.

XIX GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN

The final round. First day. 9 form.
Ratmino. 2023. July 30.

1. The ratio of the medianAM of a triangle ABC to the side BC equals $\sqrt{3} : 2$. The points on the sides of ABC dividing these side into 3 equal parts are marked. Prove that some 4 of these 6 points are concyclic.

2. Can a regular triangle be placed inside a regular hexagon in such a way that all vertices of the triangle were seen from each vertex of the hexagon? (*Point A is seen from B, if the segment AB dots not contain internal points of the triangle.*)

3. Points A_1, A_2, B_1, B_2 lie on the circumcircle of a triangle ABC in such a way that $A_1B_1 \parallel AB$, $A_1A_2 \parallel BC$, $B_1B_2 \parallel AC$. The line AA_2 and CA_1 meet at point A', and the lines BB_2 and CB_1 meet at point B'. Prove that all lines $A'B'$ concur.

4. The incircle ω of a triangle ABC centered at I touches BC at point D. Let P be the projection of the orthocenter of ABC to the median from A. Prove that the circle AIP and ω cut off equal chords on AD.

XIX GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN

The final round. Second day. 9 form.

Ratmino. 2023. July 31.

5. A point D lie on the lateral side BC of an isosceles triangle ABC. The ray AD meets the line passing through B and parallel to the base AC at point E. Prove that the tangent to the circumcicle of triangle ABD at B bisects EC.

6. Let ABC be acute-angled triangle with circumcircle Ω. Points H and M are the orthocenter and the midpoint of BC respectively. The line HM meets the circumcircle ω of triangle BHC at point $N \neq H$. Point P lies on the arc BC of ω not containing H in such a way that $\angle HMP = 90°$. The segment PM meets Ω at point Q. Points B' and C' are the reflections of A about B and C respectively. Prove that the circumcircles of triangles $AB'C'$ and PQN are tangent.

7. Let H be the orthocenter of triangle T. The sidelines of triangle T_1 pass through the midpoints of T and are perpendicular to the corresponding bisectors of T. The vertices of triangle T_2 bisect the bisectors of T. Prove that the lines joining H with the vertices of T_1 are perpendicular to the sidelines of T_2.

8. Let ABC be a triangle with $\angle A = 120°$, I be the incenter, and M be the midpoint of BC. The line passing through M and parallel to AI meets the circle with diameter BC at points E and F (A and E lie on the same semiplane with respect to BC). The line passing through E and perpendicular to FI meets AB and AC at points P and Q respectively. Find the value of $\angle PIQ$.

XIX GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN

The final round. First day. 10 form.
Ratmino. 2023. July 30.

1. Let M be the midpoint of cathetus AB of triangle ABC with right angle A. Point D lies on the median AN of triangle AMC in such a way that the angles ACD and BCM are equal. Prove that the angle DBC is also equal to these angles.

2. The Euler line of a scalene triangle touches its incircle. Prove that this triangle is obtuse-angled.

3. Let ω be the circumcircle of triangle ABC, O be its center, A' be the point of ω opposite to A, and D be a point on a minor arc BC of ω. A point D' is the reflection of D about BC. The line $A'D'$ meets ω for the second time at point E. The perpendicular bisector to $D'E$ meets AB and AC at points F and G respectively. Prove that $\angle FOG = 180^\circ - 2\angle BAC$.

4. Let ABC be a Poncelet triangle, A_1 is the reflection of A about the incenter I, A_2 is isogonally conjugated to A_1 with respect to ABC. Find the locus of points A_2.

XIX GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN

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5. The incircle of a triangle ABC touches BC at point D. Let M be the midpoint of arc BAC of the circumcircle, and P, Q be the projections of M to the external bisectors of angles B and C respectively. Prove that the line PQ bisects AD.

6. Let E be the projection of the vertex C of a rectangle $ABCD$ to the diagonal BD. Prove that the common external tangents to the circles AEB and AED meet on the circle AEC.

7. There are 43 points in the space: 3 yellow and 40 red. Any four of them are not coplanar. May the number of triangles with red vertices hooked with the triangle with yellow vertices be equal to 2023? *Yellow triangle is hooked with the red one if the boundary of the red triangle meet the part of the plane bounded by the yellow triangle at the unique point. The triangles obtained by the transpositions of vertices are identical.*

8. A triangle ABC is given. Let ω_1, ω_2, ω_3, ω_4 be circles centered at points X, Y, Z, T respectively such that each of lines BC, CA, AB cuts off on them four equal chords. Prove that the centroid of ABC divides the segment joining X and the radical center of ω_2, ω_3, ω_4 in the ratio $2:1$ from X.