XVIII GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN
Final round. First day. 8 form
Ratmino, July 31, 2022

1. (I.Kukharchuk) Let $ABCD$ be a convex quadrilateral with $\angle BAD = 2\angle BCD$ and $AB = AD$. Let P be such point that $ABCP$ is a parallelogram. Prove that $CP = DP$.

2. (A.Mardanov) Let $ABCD$ be a right-angled trapezoid and M be the midpoint of its greater lateral side CD. Circumcircles ω_1 and ω_2 of triangles BCM and AMD meet for the second time at point E. Let ED meet ω_1 at point F and FB meet AD at point G. Prove that GM bisects angle BGD.

3. (D.Reznik, A.Zaslavsky) A circle ω and a point P not lying on it are given. Let ABC be an arbitrary regular triangle inscribed into ω and A', B', C' be the projections of P to BC, CA, AB. Find the locus of centroids of triangles $A'B'C'$.

4. (A.Mardanov) Let $ABCD$ be a cyclic quadrilateral, O be its circumcenter, P be a common points of its diagonals, and M, N be the midpoints of AB and CD respectively. A circle OPM meets for the second time segments AP and BP at points A_1 and B_1 respectively and a circle OPN meets for the second time segments CP and DP at points C_1 and D_1 respectively. Prove that the areas of quadrilaterals AA_1B_1B and CC_1D_1D are equal.

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Final round. Second day. 8 form

Ratmino, August 1, 2022

5. (D.Shvetsov) An incircle of triangle ABC touches AB, BC, AC at points C_1, A_1, B_1 respectively. Let A' be the reflection of A_1 about B_1C_1; point C' is defined similarly. Lines $A'C_1$ and $C'A_1$ meet at point D. Prove that $BD \parallel AC$.

6. (A.Bremzen, A.Kulakov) Two circles meeting at points A, B and a point O lying outside them are given. Using a compass and a ruler construct a ray with origin O meeting the first circle at point C and the second one at point D in such a way that the ratio $OC : OD$ be maximal.

7. (A.Shapovalov) Ten points on a plane a such that any four of them lie on the boundary of some square. Is obligatory true that all ten points lie on the boundary of some square?

8. (I.Kukharchuk) Am isosceles trapezoid $ABCD$ ($AB = CD$) is given. A point P on its circumcircle is such that segments CP and AD meet at point Q. Let L be tha midpoint of QD. Prove that the diagonal of the trapezoid is not greater than the sum of distances from the midpoints of the lateral sides to ana arbitrary point of line PL.

XVIII GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN

Final round. First day. 9 form

Ratmino, July 31, 2022

1. (Д.Швецов) Let BH be an altitude of right-angled triangle ABC ($\angle B = 90^\circ$). An excircle of triangle ABH opposite to B, touches AB at point A_1; a point C_1 is defined similarly. Prove that $AC \parallel A_1C_1$.

2. (D.Brodsky) Lateral sidelines AB and CD of a trapezoid $ABCD$ ($AD > BC$) meet at point P. Let Q be such point of segment AD that $BQ = CQ$. Prove that the line passing through the circumcenters of triangles AQC and BQD is perpendicular to PQ.

3. (L.Emelyanov) Let circles s_1 and s_2 meet at point A and B. Consider all lines passing through A and meeting the circles for the second time at points P_1 and P_2. Construct by a compass and a ruler such line that $P_1A \cdot AP_2$ is maximal.

4. (B.Yakovlev) Let ABC be an isosceles triangle with $AB = AC$, P be the midpoint of the minor arc AB of its circumcircle, and Q be the midpoint of AC. A circumcircle of triangle APQ centered at O meets for the second time AB at point K. Prove that lines PO and KQ meet on the bisector of angle ABC.

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5. (A.Mardanov) Chords AB and CD of a circle ω meet at point E in such a way that $AD = AE = EB$. Let F be such point of segment CE that $ED = CF$. The bisector of angle AFC meets an arc DAC at point P. Prove that A, E, F, and P are concyclic.

6. (A.Mardanov) A medial line parallel to the side AC of a triangle ABC meets its circumcircle at points X and Y. Let I be tha incenter of triangle ABC and D be the midpoint of an arc AC not containing B. A point L lie on segment DI in such a way that $DL = BI/2$. Prove that $\angle IXL = \angle IYL$.

7. (I.Kukharchuk) Let H be the orthocenter of an acute-angled triangle ABC. The circumcircle of triangle AHC meets segments AB and BC at points P and Q. Lines PQ and AC meet at point R. A point K lie on the line PH in such a way that $\angle KAC = 90^\circ$. Prove that KR is perpendicular to one of medians of triangle ABC.

8. (F.Nilov) Several circles are drawn on the plane and all points of their meeting or touching are marked. May be that each circle contains exactly five marked points and exactly five marked points lie on each circle?

XVIII GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN
Final round. First day. 10 form

Ratmino, July 31, 2022

1. (Tran Quang Hung) Let $A_1A_2A_3A_4$ and $B_1B_2B_3B_4$ be two clockwisely oriented squares. The perpendicular bisectors to segments A_1B_1, A_2B_2, A_3B_3, A_4B_4 meet the perpendicular bisectors to segments A_2B_2, A_3B_3, A_4B_4, A_1B_1 at points P, Q, R, S respectively. Prove that $PR \perp QS$.

2. (A.Kuznetsov) Let $ABCD$ be a convex quadrilateral. The common external tangents to circles (ABC) and (ACD) meet at point E, the common external tangents to circles (ABD) and (BCD) meet at point F. Let F lie on AC, prove that E lie on BD.

3. (G.Chelnokov) A line meets a segment AB at point C. Which is the maximal number of points X of this line such that one of angles AXC and BXC is equal to a half of the second one?

4. (A.Matveev, I.Frolov) Let $ABCD$ be a convex quadrilateral with $\angle B = \angle D$. Prove that the midpoint of BD lies on the common internal tangent to the incircles of triangles ABC and ACD.

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Final round. Second day. 10 form

Ratmino, August 1, 2022

5. (A.Mardanov, K.Struihina.) Let AB and AC be the tangents from a point A to a circle Ω. Let M be the midpoint of BC and P be an arbitrary point on this segment. A line AP meets Ω at points D and E. Prove that the common external tangents to circles MDP and MPE meet on the medial line of triangle ABC.

6. (D.Brodsky) Let O, I be the circumcenter and the incenter of triangle ABC; P be an arbitrary point on segment OI; P_A, P_B, and P_C be the second common points of lines PA, PB, and PC with the circumcircle of triangle ABC. Prove that the bisectors of angles BP_AC, CP_BA, and AP_CB concur at a point lying on OI.

7. (F.Nilov) Several circles are drawn on the plane and all points of their meeting or touching are marked. May be that each circle contains exactly four marked points and exactly four marked points lie on each circle?

8. (A.Erdnigor) Let $ABCA'B'C'$ be a centrosymmetric octahedron (vertices A and A', B and B', C and C' are opposite) such that the sums of four planar angles equal 240° for each vertex. The Torricelli points T_1 and T_2 of triangles ABC and $A'BC$ are marked. Prove that the distances from T_1 and T_2 to BC are equal.