# XVII GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN

## Final round. First day. 8 form

*Povedniki, July 31, 2021*

1. (B.Frenkin) Let $ABCD$ be a convex quadrilateral. The circumcenter and the incenter of triangle $ABC$ coincide with the incenter and the circumcenter of triangle $ADC$ respectively. It is known that $AB = 1$. Find the remaining sidelengths and the angles of $ABCD$.

2. (P.Kozhevnikov) Three parallel lines $l_a$, $l_b$, $l_c$ pass through the vertices of triangle $ABC$. A line $a$ is the reflection of altitude $AH_a$ about $l_a$. Lines $b$, $c$ are defined similarly. Prove that $a$, $b$, $c$ are concurrent.

3. (A.Zaslavsky) Three cockroaches run along a circle in the same direction. They start simultaneously from a point $S$. Cockroach $A$ runs twice as slow than $B$, and three times as slow than $C$. Points $X$, $Y$ on segment $SC$ are such that $SX = XY = YC$. The lines $AX$ and $BY$ meet at point $Z$. Find the locus of centroids of triangles $ZAB$.

4. (I.Kukharchuk) Let $A_1$ and $C_1$ be the feet of altitudes $AH$ and $CH$ of an acute-angled triangle $ABC$. Points $A_2$ and $C_2$ are the reflections of $A_1$ and $C_1$ about $AC$. Prove that the distance between the circumcenters of triangles $C_2HA_1$ and $C_1HA_2$ equals $AC$.

# XVII GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN

## Final round. Second day. 8 form

*Povedniki, August 1, 2021*

5. (M.Saghafian) Points $A_1$, $A_2$, $A_3$, $A_4$ are not concyclic, the same for points $B_1$, $B_2$, $B_3$, $B_4$. For all $i$, $j$, $k$ the circumradii of triangles $A_iA_jA_k$ and $B_iB_jB_k$ are equal. Can we assert that $A_iA_j = B_iB_j$ for all $i$, $j$?

6. (M.Didin) Let $ABC$ be an acute-angled triangle. Point $P$ is such that $AP = AB$ and $PB \parallel AC$. Point $Q$ is such that $AQ = AC$ and $CQ \parallel AB$. Segments $CP$ and $BQ$ meet at point $X$. Prove that the circumcenter of triangle $ABC$ lies on the circle $(PXQ)$.

7. (I.Kulharchuk) Let $ABCDE$ be a convex pentagon such that angles $CAB$, $BCA$, $ECD$, $DEC$ and $AEC$ are equal. Prove that $CE$ bisects $BD$.

8. (S.Berlov) Does there exist a convex polygon such that all its sidelengths are equal and all triangle formed by its vertices are obtuse-angled?

# XVII GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN
## Final round. First day. 9 form

*Povedniki, July 31, 2021*

1. (F.Ivlev, A.Mardanov) Three cevians concur at a point lying inside a triangle. The feet of these cevians divide the sides into six segments, and the lengths of these segments form (in some order) a geometric progression. Prove that the lengths of the cevians also form a geometric progression.

2. (M.Volchkevich) A cyclic pentagon is given. Prove that the ratio of its square to the sum of the diagonals is not greater than the quarter of the circumradius.

3. (M.Didin, I.Frolov) Let $ABC$ be an acute-angled scalene triangle and $T$ be a point inside it such that $\angle ATB = \angle BTC = 120^\circ$. A circle centered at point $E$ passes through the midpoints of the sides of $ABC$. For $B$, $T$, $E$ collinear find angle $ABC$.

4. (M.Saghafian) Define the distance between two triangles to be the closest distance between two vertices, one from each triangle. Is it possible to draw five triangles in the plane such that for any two of them, their distance equals the sum of their circumradii?

# XVII GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN

## Final round. Second day. 9 form

*Povedniki, August 1, 2021*

5. (P.Kozhevnikov) Let $O$ be the circumcenter of triangle $ABC$. Points $X$ and $Y$ on side $BC$ are such that $AX = BX$ and $AY = CY$. Prove that the circumcircle of triangle $AXY$ passes through the circumcenters of triangles $AOB$ and $AOC$.

6. (P.Ryabov) The diagonals of trapezoid $ABCD$ ($BC \parallel AD$) meet at point $O$. Points $M$ and $N$ lie on the segments $BC$ and $AD$ respectively. The tangent to the circle $AMC$ at $C$ meets the ray $NB$ at point $P$; the tangent to the circle $BND$ at $D$ meets the ray $MA$ at point $R$. Prove that $\angle BOP = \angle AOR$.

7. (M.Didin, F.Ivlev, I.Frolov) Three sidelines of on acute-angled triangle are drawn on the plane. Fyodor wants to draw the altitudes of this triangle using a ruler and a compass. Ivan obstructs him using an eraser. For each move Fyodor may draw one line through two marked points or one circle centered at a marked point and passing through another marked point. After this Fyodor may mark an arbitrary number of points (the common points of drawn lines, arbitrary points on the drawn lines or arbitrary points on the plane). For each move Ivan erases at most three of marked point. (Fyodor may not use the erased points in his constructions but he may mark them for the second time). They move by turns, Fydors begins. Initially no points are marked. Can Fyodor draw the altitudes?

8. (A.Dadgarnia) A quadrilateral $ABCD$ is circumscribed around a circle $\omega$ centered at $I$. Lines $AC$ and $BD$ meet at point $P$, lines $AB$ and $CD$ meet at point $E$, lines $AD$ and $BC$ meet at point $F$. Point $K$ on the circumcircle of triangle $EIF$ is such that $\angle IKP = 90°$. The ray $PK$ meets $\omega$ at point $Q$. Prove that the circumcircle of triangle $EQF$ touches $\omega$.

# XVII GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN
## Final round. First day. 10 form

*Povedniki, July 31, 2021*

1. (D.Shvetsov) Let $CH$ be an altitude of right-angled triangle $ABC$ ($\angle C = 90^\circ$), $HA_1, HB_1$ be the bisectors of angles $CHB$, $AHC$ respectively, and $E$, $F$ be the midpoints of $HB_1$ and $HA_1$ respectively. Prove that the lines $AE$ and $BF$ meet on the bisector of angle $ACB$.

2. (A.Zaslavsky) Let $ABC$ be a scalene triangle, and $A_0$, $B_0$, $C_0$ be the midpoints of $BC$, $CA$, $AB$ respectively. The bisector of angle $C$ meets $A_0C_0$ and $B_0C_0$ at points $B_1$ and $A_1$ respectively. Prove that the lines $AB_1$, $BA_1$ and $A_0B_0$ concur.

3. (K.Knop, G.Chelnokov) The bisector of angle $A$ of triangle $ABC$ ($AB > AC$) meets its circumcircle at point $P$. The perpendicular to $AC$ from $C$ meets the bisector of angle $A$ at point $K$. A circle with center $P$ and radius $PK$ meets the minor arc $PA$ of the circumcircle at point $D$. Prove that the quadrilateral $ABDC$ is circumscribed.

4. (T.Korchyomkina) Can a triangle be a development of a quadrangular pyramid?

# XVII GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN

## Final round. Second day. 10 form

*Povedniki, August 1, 2021*

5. (P.Kozhevnikov) A secant meets one circle at points $A_1$, $B_1$, this secant meets a second circle at points $A_2$, $B_2$. Another secant meets the first circle at points $C_1$, $D_1$ and meets the second circle at points $C_2$, $D_2$. Prove that points $A_1C_1 \cap B_2D_2$, $A_1C_1 \cap A_2C_2$, $A_2C_2 \cap B_1D_1$, $B_2D_2 \cap B_1D_1$ lie on a circle coaxial with two given circles.

6. (D.Brodsky) The lateral sidelines $AB$ and $CD$ of trapezoid $ABCD$ meet at point $S$. Points $X$, $Y$ lie on the bisector of angle $S$ in such a way that $\angle AXC - \angle AYC = \angle ASC$. Prove that $\angle BXD - \angle BYD = \angle BSD$.

7. (M.Etesamifard) Let $I$ be the incenter of a right-angled triangle $ABC$, and $M$ be the midpoint of hypothenuse $AB$. The tangent to the circumcircle of $ABC$ at $C$ meets the line passing through $I$ and parallel to $AB$ at point $P$. Let $H$ be the orthocenter of triangle $PAB$. Prove that lines $CH$ and $PM$ meet at the incircle of triangle $ABC$.

8. (M.Didin) On the attraction "Merry parking" the auto has only two positions of a steering wheel: "right" and "strongly right" . So the auto can move along an arc with radius $r_1$ or $r_2$. The auto started from a point $A$ to the Nord, it covered the distance $l$ and rotated to the angle $\alpha < 2\pi$. Find the locus of its possible endpoints.