XIII GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN
The correspondence round

Below is the list of problems for the first (correspondence) round of the XIII Sharygin Geometrical Olympiad.

The olympiad is intended for high-school students of four eldest grades. In Russian school, these are 8-11. In the list below, each problem is indicated by the numbers of Russian school grades, for which it is intended. Foreign students of the last grade have to solve the problems for 11th grade, students of the preceding grade solve the problems for 10th grade etc. However, the participants may solve problems for elder grades as well (solutions of problems for younger grades will not be considered).

A complete solution of each problem costs 7 points. A partial solution costs from 1 to 6 points. A text without significant advancement costs 0 points. The result of a participant is the sum of all obtained marks.

In your work, please start the solution for each problem in a new page. First write down the statement of the problem, and then the solution. Present your solutions in detail, including all necessary arguments and calculations. Provide all necessary figures of sufficient size. If a problem has an explicit answer, this answer must be presented distinctly. Please, be accurate to provide good understanding and correct estimating of your work !

If your solution depends on some well-known theorems from standard textbooks, you may simply refer to them instead of providing their proofs. However, any fact not from the standard curriculum should be either proved or properly referred (with an indication of the source).

You may note the problems which you liked most (this is not obligatory). Your opinion is interesting for the Jury.

The solutions for the problems (in Russian or in English) must be delivered not earlier than on January 8, 2017 and not later than on April 1, 2017. To upload your work, enter the site **http://geometry.ru/olimp/olimpsharygin.php** and follow the instructions.
Attention: The solutions must be contained in pdf, doc or jpg files. We recommend to prepare the paper using computer or to scan it rather than to photograph it. *In the last two cases, please check readability of the file before uploading.*

If you have any technical problems with uploading of the work, apply to **geomolymp@mccme.ru** **(DON'T SEND your work to this address)**.

Winners of the correspondence round, the students of three grades before the last grade, will be invited to the final round held in Summer 2017 in Moscow region. (For instance, if the last grade is 12 then we invite winners from 9, 10, and 11 grade.) The students of the last grade, winners of the correspondence round, will be awarded by diplomas of the Olympiad. The list of the winners will be published on **www.geometry.ru** at the end of May 2017 at latest. If you want to know your detailed results, please use e-mail **geomolymp@mccme.ru**.

1. (8) Mark on a cellular paper four nodes forming a convex quadrilateral with the sidelengths equal to four different primes.

2. (8) A circle cuts off four right-angled triangles from rectangle $ABCD$. Let A_0, B_0, C_0 and D_0 be the midpoints of the correspondent hypotenuses. Prove that $A_0C_0 = B_0D_0$.

3. (8) Let I be the incenter of triangle ABC; H_B, H_C the orthocenters of triangles ACI and ABI respectively; K the touching point of the incircle with the side BC. Prove that H_B, H_C and K are collinear.

4. (8) A triangle ABC is given. Let C' be the vertex of an isosceles triangle ABC' with $\angle C' = 120°$ constructed on the other side of AB than C, and B' be the vertex of an equilateral triangle ACB' constructed on the same side of AC as ABC. Let K be the midpoint of BB'. Find the angles of triangle KCC'.

5. A segment AB is fixed on the plane. Consider all acute-angled triangles with side AB. Find the locus of

   а) (8) the vertices of their greatest angles;

   б) (8–9) their incenters.

6. (8–9) Let $ABCD$ be a convex quadrilateral with $AC = BD = AD$; E and F the midpoints of AB and CD respectively; O the common point of the diagonals. Prove that EF passes through the touching points of the incircle of triangle AOD with AO and OD.

7. (8–9) The circumcenter of a triangle lies on its incircle. Prove that the ratio of its greatest and smallest sides is less than two.

8. (8–9) Let AD be the base of trapezoid $ABCD$. It is known that the circumcenter of triangle ABC lies on BD. Prove that the circumcenter of triangle ABD lies on AC.

9. (8–9) Let C_0 be the midpoint of hypotenuse AB of triangle ABC; AA_1, BB_1 the bisectors of this triangle; I its incenter. Prove that the lines C_0I and A_1B_1 meet on the altitude from C.

10. (8–10) Points K and L on the sides AB and BC of parallelogram $ABCD$ are such that $\angle AKD = \angle CLD$. Prove that the circumcenter of triangle BKL is equidistant from A and C.

11. (8–11) A finite number of points is marked on the plane. Each three of them are not collinear. A circle is circumscribed around each triangle with marked vertices. Is it possible that all centers of these circles are also marked?

12. (9–10) Let AA_1, CC_1 be the altitudes of triangle ABC, B_0 the common point of the altitude from B and the circumcircle of ABC; and Q the common point of the circumcircles of ABC and $A_1C_1B_0$, distinct from B_0. Prove that BQ is the symmedian of ABC.

13. (9–11) Two circles pass through points A and B. A third circle touches both these circles and meets AB at points C and D. Prove that the tangents to this circle at these points are parallel to the common tangents of two given circles.

14. (9–11) Let points B and C lie on the circle with diameter AD and center O on the same side of AD. The circumcircles of triangles ABO and CDO meet BC at points F and E respectively. Prove that $R^2 = AF \cdot DE$, where R is the radius of the given circle.

15. (9–11) Let ABC be an acute-angled triangle with incircle ω and incenter I. Let ω touch AB, BC and CA at points D, E, F respectively. The circles ω_1 and ω_2 centered at J_1 and J_2 respectively are inscribed into $ADIF$ and $BDIE$. Let J_1J_2 intersect AB at point M. Prove that CD is perpendicular to IM.

16. (9–11) The tangents to the circumcircle of triangle ABC at A and B meet at point D. The circle passing through the projections of D to BC, CA, AB, meet AB for the second time at point C'. Points A', B' are defined similarly. Prove that AA', BB', CC' concur.

17. (9–11) Using a compass and a ruler, construct a point K inside an acute-angled triangle ABC so that $\angle KBA = 2\angle KAB$ and $\angle KBC = 2\angle KCB$.

18. (9–11) Let L be the common point of the symmedians of triangle ABC, and BH be its altitude. It is known that $\angle ALH = 180° - 2\angle A$. Prove that $\angle CLH = 180° - 2\angle C$.

19. (10–11) Let cevians AA', BB' and CC' of triangle ABC concur at point P. The circumcircle of triangle $PA'B'$ meets AC and BC at points M and N respectively, and the circumcircles of triangles $PC'B'$ and $PA'C'$ meet AC and BC for the second time respectively at points K and L. The line c passes through the midpoints of segments MN and KL. The lines a and b are defined similarly. Prove that a, b and c concur.

20. (10–11) Given a right-angled triangle ABC and two perpendicular lines x and y passing through the vertex A of its right angle. For an arbitrary point X on x define y_B and y_C as the reflections of y about XB and XC respectively. Let Y be the common point of y_b and y_c. Find the locus of Y (when y_b and y_c do not coincide).

21. (10–11) A convex hexagon is circumscribed about a circle of radius 1. Consider the three segments joining the midpoints of its opposite sides. Find the greatest real number r such that the length of at least one segment is at least r.

22. (10–11) Let P be an arbitrary point on the diagonal AC of cyclic quadrilateral $ABCD$, and PK, PL, PM, PN, PO be the perpendiculars from P to AB, BC, CD, DA, BD respectively. Prove that the distance from P to KN is equal to the distance from O to ML.

23. (10–11) Let a line m touch the incircle of triangle ABC. The lines passing through the incenter I and perpendicular to AI, BI, CI meet m at points A', B', C' respectively. Prove that AA', BB' and CC' concur.

24. (11) Two tetrahedrons are given. Each two faces of the same tetrahedron are not similar, but each face of the first tetrahedron is similar to some face of the second one. Does this yield that these tetrahedrons are similar?