IV RUSSIAN GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN

The final round. Grade 8. First day

1. Does a convex quadrilateral without parallel sidelines exist such that it can be divided into four equal triangles?
2. Given right triangle ABC with hypothenuse AC and $\angle A = 50^\circ$. Points K and L on the cathetus BC are such that $\angle KAC = \angle LAB = 10^\circ$. Determine the ratio CK/LB.
3. Two opposite angles of a convex quadrilateral with perpendicular diagonals are equal. Prove that a circle can be inscribed in this quadrilateral.
4. Let CC_0 be a median of triangle ABC; the medial perpendiculars to AC and BC intersect CC_0 in points A', B'; C_1 is the meet of lines AA' and BB'. Prove that $\angle C_1CA = \angle C_0CB$.
5. Given two triangles ABC, $A'B'C'$. Denote by α the angle between the altitude and the median from vertex A of triangle ABC. Angles β, γ, α', β', γ' are defined similarly. It is known that $\alpha = \alpha'$, $\beta = \beta'$, $\gamma = \gamma'$. Can we conclude that the triangles are similar?

Grade 8. Second day

6. Consider the triangles such that all their vertices are vertices of a given regular 2008-gon. What triangles are more numerous among them: acute-angled or obtuse-angled?
7. Given isosceles triangle ABC with base AC and $\angle B = \alpha$. The arc AC constructed outside the triangle has angular measure equal to β. Two lines passing through B divide the segment and the arc AC into three equal parts. Find the relation α/β.
8. A convex quadrilateral was drawn on the blackboard. Boris marked the centers of four excircles each touching one side of the quadrilateral and the extensions of two adjacent sides. After this, Alexey erased the quadrilateral. Can Boris define its perimeter?

Grade 9. First day

1. A convex polygon can be divided into 2008 equal quadrilaterals. Is it true that this polygon has a center or an axis of symmetry?

2. Given quadrilateral $ABCD$. Find the locus of points such that their projections to the lines AB, BC, CD, DA form a quadrilateral with perpendicular diagonals.

3. Prove the inequality

$$\frac{1}{\sqrt{2}\sin A} + \frac{1}{\sqrt{2}\sin B} + \frac{1}{\sqrt{2}\sin C} \leq \sqrt{\frac{p}{r}},$$

   where p and r are the semiperimeter and the inradius of triangle ABC.

4. Let CC_0 be a median of triangle ABC; the medial perpendiculars to AC and BC intersect CC_0 in points A', B'; C_1 is the common point of AA' and BB'. Points A_1, B_1 are defined similarly. Prove that circle $A_1B_1C_1$ passes through the circumcenter of triangle ABC.

5. Can the surface of a regular tetrahedron be glued over with equal regular hexagons?

Grade 9. Second day

6. Construct the triangle, given its centroid and the feet of an altitude and a bisector from the same vertex.

7. The circumradius of triangle ABC is equal to R. Another circle with the same radius passes through the orthocenter H of this triangle and intersect its circumcirle in points X, Y. Point Z is the fourth vertex of parallelogram $CXZY$. Find the circumradius of triangle ABZ.

8. Points P, Q lie on the circumcircle ω of triangle ABC. The medial perpendicular l to PQ intersects BC, CA, AB in points A', B', C'. Let A", B", C" be the second common points of l with the circles $A'PQ$, $B'PQ$, $C'PQ$. Prove that AA", BB", CC" concur.

Grade 10. First day

1. An inscribed and circumscribed n-gon is divided by some line into two inscribed and circumscribed polygons with different numbers of sides. Find n.
2. (A.Myakishev) Let triangle $A_1B_1C_1$ be symmetric to ABC wrt the incenter of its medial triangle. Prove that the orthocenter of $A_1B_1C_1$ coincide with the circumcenter of triangle formed by the excenters of ABC.
3. X and Y are the common points of two circles ω_1 and ω_2. The third circle ω touches internally ω_1 and ω_2 in P and Q respectively. Segment XY intersects ω in points M and N. The rays PM and PN intersect ω_1 in points A and D; the rays QM and QN intersect ω_2 in points B and C respectively. Prove that $AB = CD$.
4. Given three points C_0, C_1, C_2 on the line l. Find the locus of incenters of triangles ABC, such that points A, B lie on l, and the feet of the median, the bisectrix and the altitude from C coincide with C_0, C_1, C_2.
5. The section of a regular tetragonal pyramid is regular pentagon. Find the relation of its side to the side of base of pyramid.

Grade 10. Second day

6. (B.Frenkin) The product of two sides in the triangle is equal to $8Rr$, where R and r are the circumradius and the inradius of triangle. Prove that the angle between these sides is less than $60°$.
7. Two arcs with equal degree measure are constructed on the medians AA' and BB' of triangle ABC to the side of vertex C. Prove that the common chord of respective circles pass through C.
8. Given a set of points on the plane. It is known that from any three its points there exist two the distance between which isn't more than 1. Prove that this set can be divided into three parts with the diameter of each of these parts not more than 1.