XVI GEOMETRICAL OLYMPIAD IN HONOUR OF I.F.SHARYGIN
The correspondence round

Below is the list of problems for the first (correspondence) round of the XVI Sharygin Geometrical Olympiad.

The olympiad is intended for high-school students of four eldest grades. In Russian school, these are 8-11. In the list below, each problem is indicated by the numbers of Russian school grades, for which it is intended. Foreign students of the last grade have to solve the problems for 11th grade, students of the preceding grade solve the problems for 10th grade etc. However, the participants may solve problems for elder grades as well (solutions of problems for younger grades will not be considered).

A complete solution of each problem costs 7 points. A partial solution costs from 1 to 6 points. A text without significant advancement costs 0 points. The result of a participant is the sum of all obtained marks.

First write down the statement of the problem, and then the solution. Present your solutions in detail, including all necessary arguments and calculations. Provide all necessary figures of sufficient size. **If a problem has an explicit answer, this answer must be presented distinctly**. Please, be accurate to provide good understanding and correct estimating of your work !

If your solution depends on some well-known theorems from standard textbooks, you may simply refer to them instead of providing their proofs. However, any fact not from the standard curriculum should be either proved or properly referred (with an indication of the source).

You may note the problems which you liked most (this is not obligatory). Your opinion is interesting for the Jury.

The **solutions** for the problems (in Russian or in English) must be **delivered not before December 1, 2019 and not later than on March 1, 2020**. To upload your work, enter the site **https://contest.yandex.ru/geomshar/**, indicate the language (English) in the right upper part of the page, press "Registration"in the left upper part, and follow the instructions.

Attention:

1. The solution of each problem (and of each part of it if any) must be contained in a **separate** pdf, doc, docx or jpg file. If the solution is contained in several files then pack them to an archive (zip or rar) and load it.

2. We recommend to prepare the paper using computer or to scan it rather than to photograph it. **In all cases, please check readability of the file before uploading**.

3. If you upload the solution of some problem more than once then only the last version is retained in the checking system. **Thus if you need to change something in your solution then you have to upload the whole solution again.**

If you have any technical problems with uploading of the work, apply to **geomshar@yandex.ru (DON'T SEND your work to this address)**.

The final round will be held in July–August 2020 in Moscow region. The winners of the correspondence round are invited to it if they don't graduate from school before the round. The cutoff is determined after the examination of the papers of the correspondence round according to the number of participants with any given score. The graduates who are winners of the correspondence round will be awarded by diplomas of the Olympiad. The list of the winners will be published on **www.geometry.ru** at the end of May 2020 at latest. If you want to know your detailed results, please use e-mail **geomshar@yandex.ru**.

1. (8) Let ABC be a triangle with $\angle C = 90°$, and A_0, B_0, C_0 be the midpoints of sides BC, CA, AB respectively. Two regular triangles AB_0C_1 and BA_0C_2 are constructed outside ABC. Find the angle $C_0C_1C_2$.

2. (8) Let $ABCD$ be a cyclic quadrilateral. A circle passing through A and B meets AC and BD at points E and F respectively. The lines AF and BC meet at point P, and the lines BE and AD meet at point Q. Prove that PQ is parallel to CD.

3. (8) Let ABC be a triangle with $\angle C = 90°$, and D be a point outside ABC, such that $\angle ADC = \angle BAC$. The segments CD and AB meet at point E. It is known that the distance from E to AC is equal to the circumradius of triangle ADE. Find the angles of triangle ABC.

4. (8) Let $ABCD$ be an isosceles trapezoid with bases AB and CD. Prove that the centroid of triangle ABD lies on line CF, where F is the projection of D to AB.

5. (8–9) Let BB_1, CC_1 be the altitudes of triangle ABC, and AD be the diameter of its circumcircle. The lines BB_1 and DC_1 meet at point E, the lines CC_1 and DB_1 meet at point F. Prove that $\angle CAE = \angle BAF$.

6. (8–9) Circles ω_1 and ω_2 meet at points P and Q. Let O be the common point of common external tangents to ω_1 and ω_2. A line passing trough O meets ω_1 and ω_2 respectively at points A and B located on the same side with respect to the line PQ. The line PA meets ω_2 for the second time at C, and the line QB meets ω_1 for the second time at D. Prove that O, C, and D are collinear.

7. (8–9) Prove that the medial lines of triangle ABC meet the sides of triangle formed by its excenters at six concyclic points.

8. (8–9) Two circles meeting at points P and R are given. Let l_1, l_2 be two lines passing through P. The line l_1 meets the circles for the second time at points A_1 and B_1. The tangents at these points to the circumcircle of triangle A_1RB_1 meet at point C_1. The line C_1R meets A_1B_1 at point D_1. Points A_2, B_2, C_2, D_2 are defined similarly. Prove that the circles D_1D_2P and C_1C_2R touch.

9. (8–9) The vertex A, the circumcenter O, and the Euler line l of triangle ABC are given. It is known that l meets AB and AC at two points equidistant from A. Restore the triangle.

10. (8–9) Given are a closed broken line $A_1A_2 \dots A_n$ and a circle ω which touches each of lines $A_1A_2, A_2A_3, \dots, A_nA_1$. Call the link *good*, if it touches ω, and *bad* otherwise (i.e. if the extension of this link touches ω). Prove that the number of bad links is even.

11. (8–9) Let ABC be a triangle with $\angle A = 60°$, AD be its bisector, and PDQ be a regular triangle with altitude DA. The lines PB and QC meet at point K. Prove that AK is a symmedian of ABC.

12. (8–10) Let H be the orthocenter of a nonisosceles triangle ABC. The bisector of angle BHC meets AB and AC at points P and Q respectively. The perpendiculars to AB and AC from P and Q meet at K. Prove that KH bisects the segment BC.

13. (9–11) Let I be the incenter of triangle ABC. The excircle with center I_A touches the side BC at A'. The line l passing through I and perpendicular to BI meets I_AA' at point K lying on the medial line parallel to BC. Prove that $\angle B \leq 60°$.

14. (9–11) A nonisosceles triangle is given. Prove that one of the circles touching internally its incircle and circumcircle and touching externally one of its excircles passes through a vertex of the triangle.

15. (9–11) A circle passing through the vertices B and D of quadrilateral $ABCD$ meets AB, BC, CD, and DA at points K, L, M, and N respectively. A circle passing through K and M meets AC at P and Q. Prove that L, N, P, and Q are concyclic.

16. (9–11) Cevians AP and AQ of a triangle ABC are symmetric with respect to its bisector. Let X, Y be the projections of B to AP and AQ respectively, and N, M be the projections of C to AP and AQ respectively. Prove that XM and NY meet on BC.

17. (10–11) Chords A_1A_2 and B_1B_2 meet at point D. Suppose D' is the inversion image of D and the line A_1B_1 meets the perpendicular bisector to DD' at a point C. Prove that $CD \parallel A_2B_2$.

18. (10–11) Bisectors AA_1, BB_1, and CC_1 of triangle ABC meet at point I. The perpendicular bisector to BB_1 meets AA_1, CC_1 at points A_0, C_0 respectively. Prove that the circumcircles of triangles A_0IC_0 and ABC touch.

19. (10–11) Quadrilateral $ABCD$ is such that $AB \perp CD$ and $AD \perp BC$. Prove that there exists a point such that the distances from it to the sidelines are proportional to the lengths of the corresponding sides.

20. (10–11) The line touching the incircle of triangle ABC and parallel to BC meets the external bisector of angle A at point X. Let Y be the midpoint of arc BAC of the circumcircle. Prove that the angle XIY is right.

21. (10–11) The diagonals of bicentric quadrilateral $ABCD$ meet at point L. Given are three segments equal to AL, BL, CL. Restore the quadrilateral using a compass and a ruler.

22. (10–11) Let Ω be the circumcircle of cyclic quadrilateral $ABCD$. Consider such pairs of points P, Q of diagonal AC that the rays BP and BQ are symmetric with respect the bisector of angle B. Find the locus of circumcenters of triangles PDQ.

23. (10–11) A non-self-intersecting polygon is *nearly convex* if precisely one of its interior angles is greater than 180°.

    One million distinct points lie in the plane in such a way that no three of them are collinear. We would like to construct a nearly convex one-million-gon whose vertices are precisely the one million given points. Is it possible that there exist precisely ten such polygons?

24. (11) Let I be the incenter of a tetrahedron $ABCD$, and J be the center of the exsphere touching the face BCD and the planes containing three remaining faces (outside these faces). The segment IJ meets the circumsphere of the tetrahedron at point K. Which of two segments IK and JK is longer?