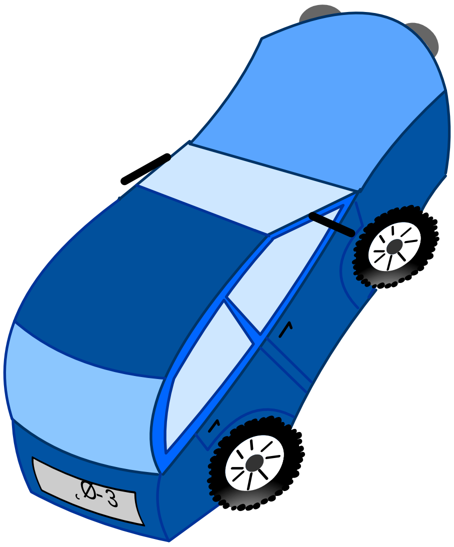
III I.F.Sharygin olympiad in geometry

Final round. 8 form

1. Determine on which side is the steering wheel disposed in the car depicted in the figure.



2. By straightedge and compass, reconstruct a right triangle ABC ($\angle C = 90°$), given the vertices A, C and a point on the bisector of angle B.

3. The diagonals of a convex quadrilateral dissect it into four similar triangles. Prove that this quadrilateral can also be dissected into two congruent triangles.

4. Determine the locus of orthocenters of triangles, given the midpoint of a side and the bases of the altitudes drawn to two other sides.

5. Medians AA' and BB' of triangle ABC meet at point M, and $\angle AMB = 120°$. Prove that angles $AB'M$ and $BA'M$ are neither both acute nor both obtuse.

6. Two non-congruent triangles are called *analogous* if they can be denoted as ABC and $A'B'C'$ such that $AB = A'B'$, $AC = A'C'$ and $\angle B = \angle B'$. Do there exist three mutually analogous triangles?

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Final round. 9 form

1. Given a circumscribed quadrilateral $ABCD$. Prove that its inradius is smaller than the sum of the inradii of triangles ABC and ACD.

2. Points E and F are chosen on the base side AD and the lateral side AB of an isosceles trapezoid $ABCD$, respectively. Quadrilateral $CDEF$ is an isosceles trapezoid as well. Prove that $AE \cdot ED = AF \cdot FB$.

3. Given a hexagon $ABCDEF$ such that $AB = BC$, $CD = DE$, $EF = FA$, and $\angle A = \angle C = \angle E$. Prove that lines AD, BE, and CF are concurrent.

4. Given a triangle ABC. An arbitrary point P is chosen on the circumcircle of triangle ABH (H is the orthocenter of triangle ABC). Lines AP and BP meet the opposite sidelines of the triangle at points A' and B', respectively. Determine the locus of midpoints of segments $A'B'$.

5. Reconstruct a triangle, given the incenter, the midpoint of some side and the base of the altitude drawn to this side.

6. A cube with edge length $2n + 1$ is dissected into small cubes of size $1 \times 1 \times 1$ and bars of size $2 \times 2 \times 1$. Find the least possible number of cubes in such a dissection.

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Final round. 10 form

1. In an acute triangle ABC, altitudes at vertices A and B and bisector line at angle C intersect the circumcircle again at points A_1, B_1 and C_0. Using the straightedge and compass, reconstruct the triangle by points A_1, B_1 and C_0.

2. Points A', B', C' are the bases of the altitudes AA', BB' and CC' of an acute triangle ABC. A circle with center B and radius BB' meets line $A'C'$ at points K and L (points K and A are on the same side of line BB'). Prove that the intersection point of lines AK and CL belongs to line BO (O is the circumcenter of triangle ABC).

3. Given two circles intersecting at points P and Q. Let C be an arbitrary point distinct from P and Q on the former circle. Let lines CP and CQ intersect again the latter circle at points A and B, respectively. Determine the locus of the circumcenters of triangles ABC.

4. A quadrilateral $ABCD$ is inscribed into a circle with center O. Points C', D' are the reflections of the orthocenters of triangles ABD and ABC at point O. Lines BD and BD' are symmetric with respect to the bisector of angle ABC. Prove that lines AC and AC' are symmetric with respect to the bisector of angle DAB.

5. Each edge of a convex polyhedron is shifted such that the obtained edges form the frame of another convex polyhedron. Are these two polyhedra necessarily congruent?

6. Given are two concentric circles Ω and ω. Each of the circles b_1 and b_2 is externally tangent to ω and internally tangent to Ω, and each of the circles c_1 and c_2 is internally tangent to both Ω and ω. Mark each point where one of the circles b_1, b_2 intersects one of the circles c_1 and c_2. Prove that there exist two circles distinct from b_1, b_2, c_1, c_2 which contain all 8 marked points. (Some of these new circles may appear to be lines.))